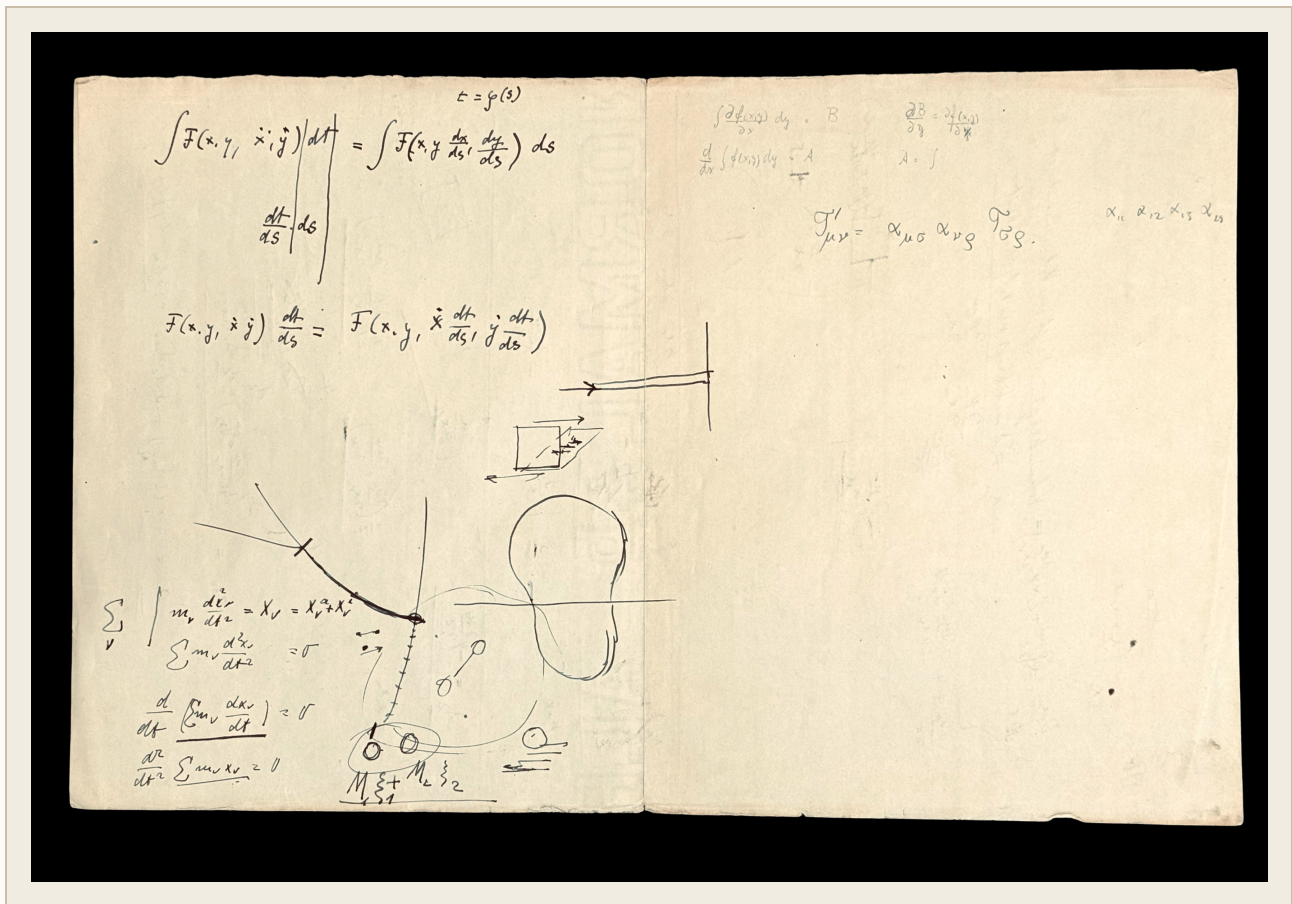


Albert Einstein

A four-panel working bifolium from Einstein's early post-Minkowski tensor phase

Undated. On notational, physical, and historical grounds, most plausibly **1912-1913**, during Einstein's early Zurich tensor period. A single large sheet, 330 × 210 mm open, scored at the fold; the exterior shows differential toning consistent with long-term folded storage.

• THE OPEN BIFOLIUM



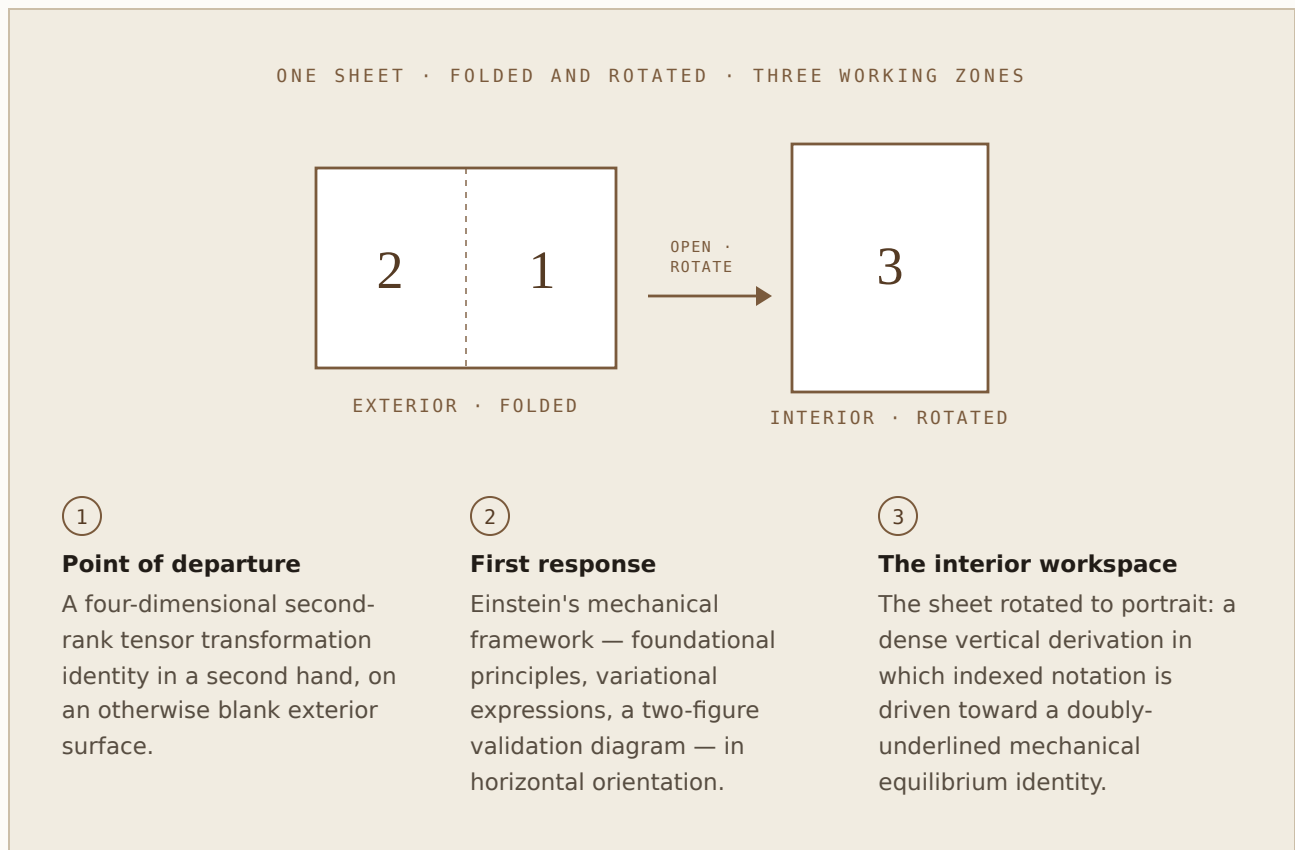
The open bifolium, both exterior panels. The tensor identity in an unidentified second hand is visible at upper centre-right.

The Manhattan Rare Book Company, ABAA, ILAB

A working sheet

After 1912, Einstein was learning to apply four-dimensional tensor methods to physical problems. This manuscript is consistent with that transition: tensor formalism tested against classical mechanics in a concentrated working session.

The notational and physical evidence points most plausibly to the 1912–13 window. Autograph scientific manuscripts by Einstein are exceptionally rare, and we find no manuscript in public auction records documenting this application of tensor calculus to matter-side mechanics: the translation of physical quantities into tensor form on which General Relativity would depend.



Post-Minkowski mechanics and the Zurich Notebook

The Zurich Notebook is the primary documentary record of Einstein's path toward General Relativity, and one of the most closely studied scientific manuscripts in modern physics. The present bifolium belongs to the same early post-Minkowski problem-space: the effort to make four-dimensional tensor formalism physically meaningful in mechanics.

In the Zurich Notebook, tensor notation and matter terms appear together within dense computational work. The present bifolium preserves a compact, structurally comparable integration on a single folded leaf: Zone 1 carries a formal four-dimensional tensor transformation identity in a hand distinct from Einstein's; Zone 2 develops the mechanical framework through *Volumenkraft*, *Druckkraft*, and diagrammatic testing; and Zone 3 applies indexed notation directly to the derivation. The heavily underlined result at the foot of Zone 3 marks a local resolution within the calculation — tensor notation producing a mechanical equilibrium condition.

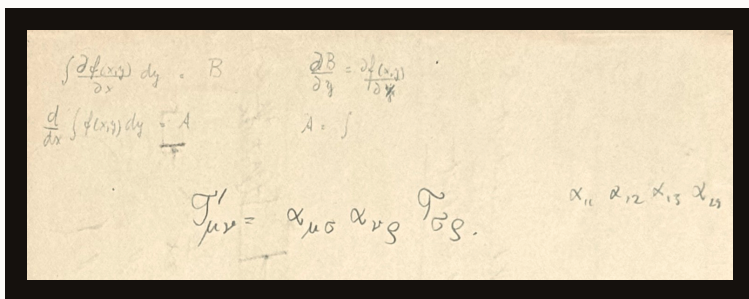
The comparison is structural, but the dating implication is stronger than a loose analogy. The sheet's combination of all-lower-index notation, explicit summation in Einstein's hand, mixed coordinate idiom, compact second-hand tensor identity, and visible effort to make formal tensor language mechanically operational converges on the 1912–13 window as the most plausible setting. Later revisitations cannot be excluded, but they explain the sheet less naturally than the early Zurich tensor period, when Einstein was newly absorbing the post-Minkowski language needed to make mechanics, matter, and gravitation speak in tensorial form — and when he was receiving mathematical guidance, especially from Marcel Grossmann, in the tensor calculus that General Relativity would require. The importance of the present manuscript is that it shows Einstein's method in detail: the movement from formal transformation language to physical terms, diagrams, local force relations, and finally a mechanical equilibrium result.

One equation, in another hand

At the head of the manuscript, in a hand distinct from Einstein's, stands a compact four-dimensional second-rank tensor transformation identity — a formal statement of precisely the kind needed to treat physical quantities relativistically. Its indexed form sets the four-dimensional framework the rest of the sheet works within. Einstein's own derivations proceed independently, working the physics of matter into indexed mechanical form.

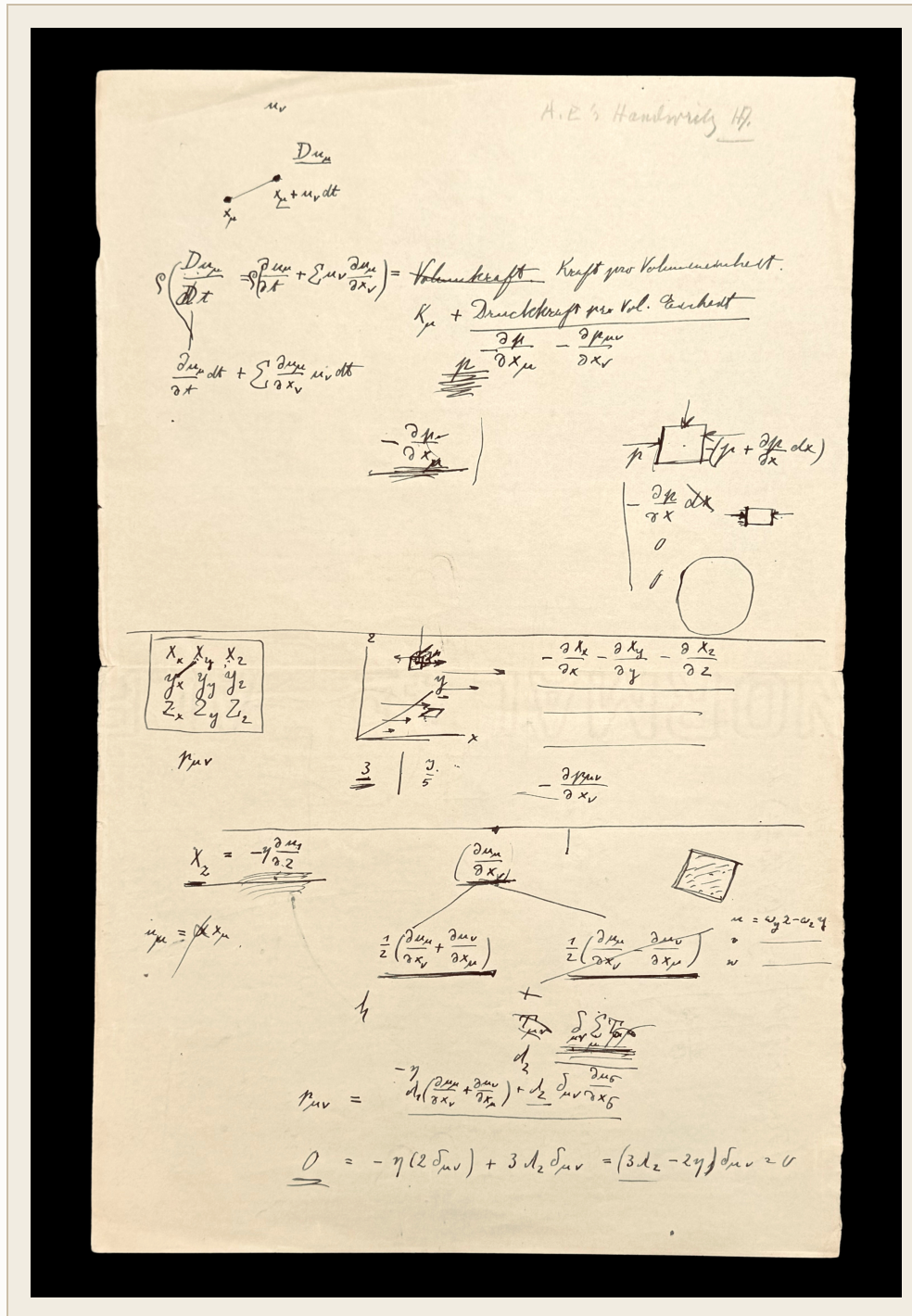
The hand is neat and deliberate, unlike Einstein's compressed working script, and remains unidentified — including against Grossmann and Ehrenfest, the natural candidates. The identity omits summation signs, while Einstein writes them out. The contrast is consistent with a formal statement supplied, copied, or noted from discussion — a point of departure followed by Einstein's effort to make tensor formalism physically operational. The combination supports an early date: 1912–13 was precisely the period when Einstein was absorbing the post-Minkowski tensor language that General Relativity would require.

• ZONE 1, DETAIL



The transformation matrix written out in full alongside the identity — $\alpha_{11} \alpha_{12} \alpha_{13} \alpha_{14}$ — its components indexed individually.

Above the identity, in pencil, a brief calculation compares differentiation and integration of a two-variable function — a local analytic check tied to the stress-gradient and force-density work elsewhere on the sheet. The pencil is fainter and quicker than the inked identity above, consistent with Einstein's hand.



Zone 3, reached by rotating the sheet as though opening a notebook: Einstein's own hand, progressively more compressed, working the mechanical relations toward a chained tensor equality, doubly underlined.

An ordinary calculation, actively worked

The calculations are not especially complex by Einstein's eventual standards — and that ordinariness is part of their importance. The sheet records tensor notation being made operational in ordinary mechanics: pressure, stress, force, and equilibrium. The graphological urgency visible across Zone 3 — the cancellations, the restarts, the accelerating script — is characteristic of active working-through rather than fair-copy transcription, consistent with a mind testing unfamiliar material rather than recording settled results.

The historical value lies less in the final equality than in the route by which Einstein reaches it. The importance of the sheet is that it preserves not a possible solution in retrospect, but Einstein's own working path through the problem: formal notation brought into contact with local mechanical quantities, diagrammatic testing, and repeated calculation until the tensor language yields a mechanical result. Precisely because the calculation is ordinary in subject, the visible effort matters: at the moment represented by this bifolium, making tensor formalism work in ordinary mechanics was not yet elementary for Einstein, but a difficult act of translation between mathematical form and physical meaning.

Einstein described this class of work in similar terms himself. In a letter to Michele Besso of 26 March 1912 (*Collected Papers of Albert Einstein*, vol. 5, no. 377), he called each step “devilishly difficult,” and the work he had so far done “certainly still the simplest of all.” That such work could be at once elementary and hard-won is characteristic of the inquiry itself.

Notational features: the early post-Minkowski window

Several features of the manuscript align with Einstein's early tensor usage as documented in the Zurich Notebook, each finding a direct parallel in the Notebook of the 1912–13 period. They read most naturally as markers of early tensor practice — consistent with an early date, though not mandating one, since Einstein returned to such notation for specific problems throughout his career. What they establish is a clear relationship to a specific phase of his working practice.

All-downstairs indices. Tensor components appear exclusively with lower indices; the covariant/contravariant distinction that became standard in Einstein's later work is absent here, as in the Zurich Notebook.

Explicit summation. Summation symbols are written out throughout, before the streamlined conventions Einstein later adopted. The identity in the unidentified second hand omits summation signs, while Einstein's working calculations write them explicitly — a contrast consistent with mixed notation in the early tensor period.

Conceptual shorthand. German terms — *Volumenkraft*, *Druckkraft* — sit alongside the tensor notation. In published work such terms give way to formal expressions; in a worksheet they act as cognitive anchors, holding the physics in view while the mathematics develops — a mark of a working document, not a finished one.

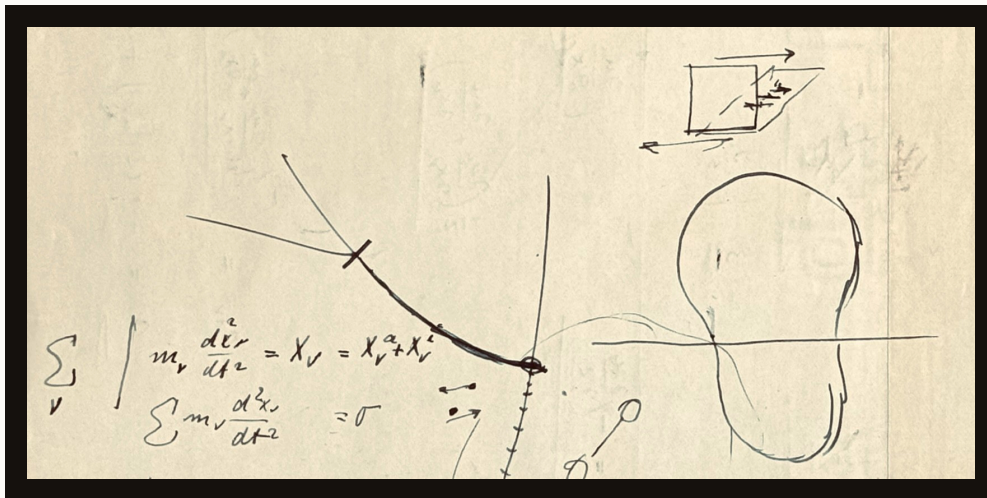
Mixed coordinate idiom. Greek indices for tensor quantities, Latin spatial coordinates (X, Y, Z) retained in the derivations — the mixed coordinate idiom of Einstein's early tensor work.

The asymmetrical test body

Embedded in the derivation are two related diagrams: a small rectangular differential element and a larger irregular closed curve. Their proximity suggests a local-to-global validation structure, testing whether stress relations derived locally remain mechanically coherent in a non-symmetric body.

A symmetric form — a sphere, a cube — can let an inconsistency between components cancel and go unseen; an arbitrary outline allows no such shortcut, so relations that survive it survive generally. Whether or not it was set down as a deliberate test, a check of this kind is the sort any formalism meant to hold under arbitrary geometry must eventually satisfy on the matter side.

• THE VALIDATION FIGURES



A small rectangular differential element beside a large, deliberately irregular closed curve — local relations derived on the regular element, then tested against a body with no exploitable symmetry.

An open question, with a working hypothesis

The dating has been assessed as most plausibly 1912–1913, though the content does not in itself fix a precise date. The manuscript's combination of four-dimensional tensor language, all-lower-index notation, explicit summation, mechanical force vocabulary, and an unidentified second hand fits most naturally with Einstein's early Zurich tensor period. Later revisitations cannot be excluded, since Einstein returned to elementary problems and older notation throughout his career, but they are less explanatory of the sheet's visible urgency, its second-hand formal identity, and its reconciliation of tensor notation with classical mechanics.

Four windows fit the physical and notational evidence:

- 1 · PRE-NOTEBOOK Post-Prague / early Zurich, 1912: a matter-side feasibility probe testing whether four-dimensional tensor notation could handle classical mechanics.
- 2 · CONCURRENT Contemporaneous with the Notebook: a parallel working sheet from the same period, treating on one leaf the mechanics-into-tensor problem the Notebook pursues at greater length.
- 3 · 1913–15 A revisitation in 1913–1915, consolidating or stress-testing earlier work during the final push toward the field equations.
- 4 · POST-1916 A later revisitation: possible, though less consistent with the sheet's apparent urgency, second-hand formal identity, and correction-heavy effort to make tensor notation mechanically operational.

Whatever the window, the sheet remains an Einstein working manuscript applying tensor notation to continuum mechanics. The preferred 1912–13 dating sharpens its significance, but its importance does not depend on a single reconstruction.

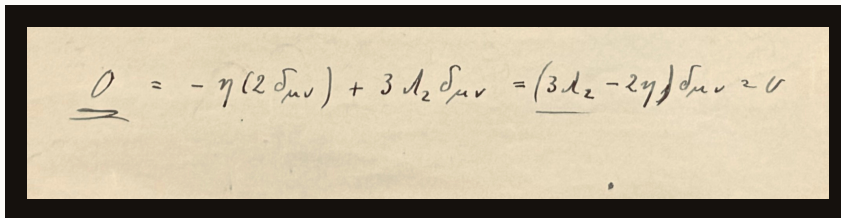
Record and history

- ATTRIBUTION** Pencil annotation by Helen Dukas, “A.E.’s Handwriting H.D.”
— consistent with preservation among Einstein’s retained research papers.
- AUCTION** Sotheby’s, New York, 26 June 1998.
- COLLECTION** Subsequently in the collection of Jagdish Mehra.

Paper

The watermark appears to identify the sheet as German *Normalpapier* from the Hannoversche Papierfabriken Alfeld-Gronau, grade “4b” — a draft or concept paper suitable for calculations and internal use.

• ZONE 3, THE RESOLUTION



The foot of Zone 3: the chained equality resolving to zero, doubly underlined — a mechanical equilibrium condition reached through indexed calculation.

This bifolium preserves tensor formalism becoming usable physics in Einstein’s hand. The sheet does not merely record a formal identity; it brings formal tensor language into contact with pressure, stress, force density, volume elements, and equilibrium. For Einstein, this was the essential post-Minkowski problem: tensor notation had to represent matter correctly if it was to serve as the language of a general theory of gravitation. Its importance lies in how he did that work — not as abstract argument, but as calculation, diagram, correction, and mechanical resolution. No manuscript known to us has appeared in public auction records documenting Einstein’s application of tensor calculus to matter-side mechanics in this form.